

# Internet Appendix

## SUPPLEMENTARY MATERIAL FOR: IDENTIFYING FIRM-LEVEL FINANCIAL FRICTIONS USING THEORY-INFORMED RESTRICTIONS (FOR ONLINE PUBLICATION)

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### **Abstract**

This supplement contains:

- S1: Proof of Proposition 1
- S2: Extended model derivations
- S3: Econometric details
- S4: Simulation Study
- S5: More empirical results

# S1 Proofs

## Proof of Proposition 1

Here we prove the sign restrictions in (16) and the magnitude restrictions in (17). From the budget constraint (2) we obtain:

$$\Theta(b_t, \xi_t) = wl_t + \text{div}_t - (s_t - \theta_t F) \quad (\text{S1})$$

The net funds borrowed,  $\Theta(b_t, \xi_t)$ , cover the difference between financial wealth net of the overhead costs,  $s_t - \theta_t F$ , and the expenditures for wages  $wl_t$  and dividends  $\text{div}_t$ . As discussed in Section 2.1, the discount term  $\nu > 1$  is an incentive for the firm to borrow to distribute dividends. It follows that in the steady state the firm is a net borrower, and  $\Theta(b_t, \xi_t)$  is strictly positive. To see this, consider a firm that is unconstrained, and is able to finance its wage bill with internal finance, without borrowing. In this case, keeping one unit of resources as savings generates  $1 + r$  units next period, and hence its net present value is  $(1 + r)(\frac{1}{1+r}\frac{1}{\nu}) = \frac{1}{\nu} < 1$ . Since the value to distribute this unit as dividends in the current period  $t$  is equal to 1, it follows that the firm finds it optimal to distribute all its savings as dividends. Then the firm will start to borrow to increase dividends further, and in doing so its cost of borrowing increases above  $1 + r$ , because of financing frictions, until it becomes so large that the firm will stop increasing dividends.

Furthermore, notice that the functional form in (4) implies that the elasticity of the amount borrowed  $\Theta(b_t, \xi_t)$  with respect to the financial frictions process  $\xi_t$  is negative. Moreover, the function  $\Theta(b_t, \xi_t)$  is concave and increasing in  $b_t$ , and reaches the maximum value for  $b_t = b_t^M \equiv \left[ \frac{1}{1+r} \frac{1}{\gamma \xi_t} \right]^{\frac{1}{\gamma-1}}$ . The firm will never optimally choose a face value of debt larger than  $b_t^M$  because it would imply increasing the promised payment next period to the creditors while actually receiving less funds today. Therefore, a necessary condition for the optimal choice of debt is:

$$\frac{\partial \Theta_t}{\partial b_t} \frac{b_t}{\Theta_t} > 0 . \quad (\text{S2})$$

We log-linearize  $\Theta(b_t, \xi_t)$  and obtain

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t , \quad (\text{S3})$$

where the above discussion clarifies that that  $\pi_b > 0$  and  $\pi_\xi > 0$ .

Before log-linearising the budget constraint, we need to define the optimal dividend policy. In equilibrium the firm decides dividends balancing the need to distribute earnings early, because of the high discount factor, with the need to save to accumulate wealth and reduce future financial frictions. Since dividends are positive in the steady state, we first consider a log-linear approximation assuming they have positive values around their steady state value, and then we show that the proof remains valid also when dividends are zero because the constraint (3) is binding. Conditional on current wealth  $s_t$ , a sufficient statistic for the opportunity cost to distribute dividends rather than saving is the shadow cost of finance  $\psi_t$ . For example, if the firm is suddenly more productive and needs to increase production inputs, this will increase  $\psi_t$  in equilibrium signalling an higher cost of distributing dividends and hence reducing available resources to invest. Therefore, conditional on current financial wealth  $s_t$ , an increase in the shadow cost of finance  $\psi_t$  implies the firm will reduce dividends. Hence we can approximate the dynamics of dividends with the following log-linear relation:

$$\log \text{div}_t = \pi_{div}^s \log s_t - \pi_{div} \log \psi_t , \quad (\text{S4})$$

where  $\pi_{div} > 0$  and  $\pi_{div}^s > 0$ . Log-linearising (S1) around the steady state and using (S3) and (S4) yields:

$$\pi_b \log b_t - \pi_\xi \log \xi_t = - \left( \frac{s - \overline{\text{div}} \pi_{div}^s}{\Theta} \right) \log s_t + \frac{F}{\Theta} \log \theta_t + \frac{wl}{\Theta} \log l_t - \overline{\text{div}} \pi_{div} \log \psi_t ,$$

where  $\overline{\text{div}}$  is the (positive) steady state value of dividends, and as discussed above,  $\Theta$  is positive. We substitute  $\log l_t$  and  $\log b_t$  using (10) and solve for  $\log \psi_t$ . The comparison with Equation (11) delivers the solution for the coefficients  $\pi_2$ ,  $\pi_3$  and  $\pi_4$ :

$$\begin{aligned} \pi_2 &= \frac{\pi_b \frac{1}{\gamma-1} + \pi_\xi}{\pi_b \frac{1}{\gamma-1} \frac{1}{\psi-1} + \frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\overline{\text{div}} \pi_{div}}{\Theta}} , & \pi_3 &= \frac{\frac{F}{\Theta}}{\pi_b \frac{1}{\gamma-1} \frac{1}{\psi-1} + \frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\overline{\text{div}} \pi_{div}}{\Theta}} \\ \text{and} \quad \pi_4 &= \frac{\frac{wl}{\Theta} \frac{1}{1-\alpha}}{\pi_b \frac{1}{\gamma-1} \frac{1}{\psi-1} + \frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\overline{\text{div}} \pi_{div}}{\Theta}} \end{aligned} \quad (\text{S5})$$

Assumptions 1 and 2 together with the fact that  $\pi_\xi$  and  $\pi_b$  are positive constants, that

$\pi_{div}$  is positive, and that  $\psi > 1$ , imply that  $\pi_2$  and  $\pi_3$  are positive coefficients, while  $\pi_4$  is between 0 and 1. This proves all the magnitude restrictions in (17) and all the sign restrictions in (16) except the fact that the element  $B_{11}$  in the matrix  $B$  is negative.

The term  $B_{11}$  is equal to  $-(\frac{1}{\gamma-1} - \frac{\pi_2}{(\gamma-1)(\psi-1)})$ .  $\frac{1}{\gamma-1}$  is the direct negative effect of the  $\xi_t$  shock on borrowing  $b_t$ . However, reducing borrowing reduces labour input, and this increases financial frictions  $\psi_t$  (see the labour first order condition 9) driving the firm to increase borrowing, and partly dampening the direct effect of the shock. The dampening factor is the term  $\frac{\pi_2}{(\gamma-1)(\psi-1)}$ . Intuitively, since the marginal product of labour has increased, the firm finds it optimal to borrow more until the higher marginal cost of external finance compensates the higher return on labour. Therefore, this equilibrium implies that borrowing is always lower than before the shock (otherwise labour input would not decrease), meaning that the optimality conditions imply that the dampening factor  $\frac{\pi_2}{(\gamma-1)(\psi-1)}$  cannot be larger than the direct effect  $\frac{1}{\gamma-1}$ , and hence the term  $B_{11}$  is negative. This proves Proposition 1 for the case in which dividends are positive. If instead dividends are equal to zero because the constraint (3) is binding, then the log-linearization of the budget constraints is the same as above except that the terms  $\overline{div}\pi_{div}^s$  and  $\overline{div}\pi_{div}$  are both equal to zero. In this case it is straightforward to see that the proof follows as before, since it is still the case that  $\pi_2$  and  $\pi_3$  are positive coefficients, while  $\pi_4$  is between 0 and 1.

## S2 Model extensions

### S2.1 Static model with endogenous convex excess cost of finance

In this section we provide a microfoundation of the convex cost of external finance assumed in (4). Consider the following simplified static version of our benchmark model. The firm borrows  $\hat{b} = \frac{b}{1+r} - c$  to finance the labour input and produce  $y = zl^\alpha$  at the beginning of the second period.  $r$  is the real interest rate while we assume the wage is  $w = 1$  and as in our main model, the term  $c$  measures the excess cost of financial constraints. The timing is as follow. Before observing  $z$ , the firm needs to borrow to be able to hire workers. Since borrowing is costly, the firm only borrows the amount necessary to pay wages, so that  $\hat{b} = l$ ,

and profits net of repaying the debt are:

$$\pi = z \hat{b}^\alpha - b \quad (\text{S6})$$

After borrowing and before actually hiring labour, the firm has  $\hat{b}$  amount of cash. It observes  $z$ , drawn from a given distribution with  $z \in [0, Z]$ , and then has two options: i) hire labour, produce, repay the debt and obtain the profit  $\pi$ . ii) default and steal a fraction  $\lambda$  of the cash. In this case we assume the lenders get zero return, because any additional cash is lost. The firm defaults if the return from defaulting is higher than producing and repaying the debt, namely if  $\lambda \hat{b} > \pi$ , while it chooses to produce and repay the debt if  $\lambda \hat{b} \leq \pi$ . Substituting  $\pi$  using Equation S6 and rearranging, we determine that the minimum productivity not to default, denoted with  $z^*$  is:

$$z^* = \frac{\lambda \hat{b} + b}{\hat{b}^\alpha}. \quad (\text{S7})$$

Ex ante, a risk neutral lender which requires an expected return of  $1+r$ , will lend the amount  $\hat{b}$  under the promise of a repayment of  $b$ , such that the expected repayment  $p(z > z^*) b$  is:

$$p(z > z^*) b = (1 + r) \hat{b}. \quad (\text{S8})$$

The excess cost of borrowing can be defined as:

$$c = \frac{b}{1 + r} - \hat{b}, \quad (\text{S9})$$

that is, the difference between the sum the firm would receive in the absence of financial imperfections  $\frac{b}{1 + r}$  and  $\hat{b}$ . Substituting  $\hat{b}$  in (S8) using (S9) yields:

$$c = \frac{b}{R^F} (1 - p(z > z^*))$$

if  $p(z > z^*)$  was constant, then  $c$  would be linear in the face value of debt  $b$ . However, from (S6) and (S7) it is easy to see that an increasing in borrowing  $\hat{b}$  increases  $z^*$ , reduces  $p(z > z^*)$  and increases  $b$ , implying that  $c$  is increasing and convex in  $b$ .

## S2.2 Derivation of the model with capital

In this section, we prove that Proposition 1 continues to hold for the linearized model with capital that was discussed in Section 2.3.1. The first order condition for debt is unchanged and the one for labour becomes

$$\frac{\alpha z_t k_t^\beta}{l_t^{1-\alpha}} = (1+r) \psi_t w . \quad (\text{S10})$$

Furthermore, as in the benchmark model,  $\log \psi_t$  can be approximated by a linear function:

$$\log \psi_t = \pi_1 \log s_t + \pi_2 \log \xi_t + \pi_3 \log \theta_t + \pi_4 \log z_t - \pi_5 \log k_t .^1 \quad (\text{S11})$$

By substituting it back into the log linearized equations and solving for the reduced form we obtain

$$Y_t = c + DW_t + Bg_t ,$$

Where  $W_t = (s_t, k_t)'$ , while the  $B$  matrix is identical to the one in the benchmark model. The main difference is that the budget constraint is now determined by Equation (20), where:

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t + \pi_k \log k_t$$

As in the benchmark case,  $\pi_b > 0$  and  $\pi_\xi > 0$ , while we do not need to impose any restriction on  $\pi_k$ . Log-linearising the budget constraint following the same procedure outlined in Appendix A1, using (21), and solving for  $\log \psi_t$  yields:

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<sup>1</sup>Note that the proof of proposition 1 does not require any assumption regarding the coefficient  $\pi_5$ . Such coefficient is likely positive, because more capital increases the assets available to the firm. However, it might be negative in certain situations if capital is subject to substantial disinvestment costs. Consider for example the extreme case that capital is fully irreversible. In this case, it could be that after a negative productivity shock the firm is forced to hold an inefficiently high amount of capital which increases the intensity of financial frictions. On the one hand, this possibility is unlikely to be empirically very relevant, and any bias would directly affect only the  $D$  matrix. On the other hand, one could allow for a more flexible relationship between  $\log \psi_t$  and  $k_t$ , and this would again only affect the matrix  $D$ , not the identification of  $B$ .

$$\begin{aligned}
\pi_2 &= \frac{\pi_b \frac{1}{\gamma-1} + \pi_\xi}{\pi_b \frac{1}{\gamma-1} \frac{1}{\psi-1} + \frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon_\psi^i}{\Theta}}; \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \frac{1}{\gamma-1} \frac{1}{\psi-1} + \frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon_\psi^i}{\Theta}}; \\
\pi_4 &= \frac{\frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\epsilon_z^i}{\Theta}}{\pi_b \frac{1}{\gamma-1} \frac{1}{\psi-1} + \frac{wl}{\Theta} \frac{1}{1-\alpha} + \frac{\overline{div}\pi_{div}}{\Theta} + \frac{\epsilon_\psi^i}{\Theta}}
\end{aligned} \tag{S12}$$

These coefficients are identical to those derived in the benchmark case (see Equation S5) except for the presence of the term  $\frac{\epsilon_\psi^i}{\Theta}$  at the denominator and of the term  $\frac{\epsilon_z^i}{\Theta}$  at the numerator of  $\pi_4$ . Both terms are weakly positive, as shown in (22), and moreover  $\epsilon_z^i$ , the elasticity of investment to productivity, is likely to be small if positive, because of capital adjustment costs, so that  $\pi_4$  continues to be smaller than 1, which is consistent with the fact that it represents a dampening factor. If it was larger than 1, it would imply that positive productivity shocks increase financial frictions so much that the firm actually wants to reduce its variable input  $l_t$ , which would violate profit maximization. It is then easy to see that the proof of Proposition 1 follows similarly to the benchmark case.

## S2.3 Models with collateralised borrowing

### Asset based borrowing

Following the specification in Section 2.3.2, in the case of asset based borrowing the firm maximizes (19) subject to (20), (21), (22) and (23). Taking the first order condition for debt, assuming  $\lambda_t^1$  is constant and log-linearising yields:

$$\log b_t = \frac{1}{\gamma-1} \frac{1}{\psi-1} \log \psi_t - \frac{1}{\gamma-1} \log \xi_t + \frac{\gamma}{\gamma-1} \log k_t .$$

The first order condition for labour is the same as (S10) and  $\log \psi_t$  is determined by (S11). We can substitute and solve for  $\log b_t$ ,  $\log l_t$  and  $\log y_t$ , and verify that the  $B$  Matrix is the same as in the benchmark model.

Therefore, it is possible to prove Proposition 1 following the same procedure outlined above. Intuitively, more capital  $k_t$  relaxes the borrowing constraint, but since it is predetermined, the only parameter affected is  $\pi_5$  the other parameters  $\pi_2, \pi_3$  and  $\pi_4$  are identical to those derived above in Section S1.

### Financial shock to the collateral value of assets

If we assume that the financial shock is a reduction in  $1 - \lambda_t^1$ , the collateral value of capital, rather than an increase in  $\xi_t$ , then the linearized first order condition for the debt becomes:

$$\log b_t = \frac{1}{\gamma - 1} \frac{1}{\psi - 1} \log \psi_t - \frac{\gamma}{\gamma - 1} \log(1 - \lambda_t^1) + \frac{\gamma}{\gamma - 1} \log k_t$$

Following the same procedure outlined above, the system becomes:

$$D_s = \begin{bmatrix} -\frac{1}{\gamma-1} \frac{1}{\psi-1} \pi_1 \\ \frac{1}{1-\alpha} \pi_1 \\ \frac{\alpha}{1-\alpha} \pi_1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -\left(\frac{\gamma}{\gamma-1} - \frac{1}{\gamma-1} \frac{1}{\psi-1} \pi_2\right) & \frac{\pi_3}{(\gamma-1)(\psi-1)} & \frac{\pi_4}{(\gamma-1)(\psi-1)} \\ -\frac{\pi_2}{1-\alpha} & -\frac{\pi_3}{1-\alpha} & \frac{1-\pi_4}{1-\alpha} \\ -\frac{\alpha\pi_2}{1-\alpha} & -\frac{\alpha\pi_3}{1-\alpha} & \frac{1-\alpha\pi_4}{1-\alpha} \end{bmatrix}.$$

Which is identical to the benchmark case except for the  $B_{11}$  coefficient, whose first term is  $-\frac{\gamma}{\gamma-1}$  instead of  $-\frac{1}{\gamma-1}$ . Notice that, in this case, the last term in Equation 12 would be  $g_t = \log(1 - \lambda_t, \theta_t, z_t)'$ . This difference does not alter the properties of the system and the proof of proposition 1 is once again identical to the one outlined above. In other words, our approach is able to capture any type of financial shocks, both those that increase the cost of credit and those that reduce its quantity.

### Models with collateralised borrowing: Earnings based borrowing

Using the new definition of financial costs in (24), the firm maximises the value function subject to

$$div_t = s_t - F\theta_t - wl_t + \frac{b_t}{1+r} - \xi_t \left[ \frac{b_t}{(1 - \lambda_t^2) \pi_t} \right]^\gamma.$$

Where  $\pi_t$  is defined in (25). Note that, also in this case, a positive shock to the cost of credit  $\xi_t$  and a negative shock to quantity of collateral  $\lambda_t^2$  have similar implications in a linear approximation of this model. Therefore we consider the shock to be  $\xi_t$  and we assume  $\lambda_t^2$  to be constant. Taking the first order condition for labour, and using (7) and the definition of  $\pi_t$  we obtain:

$$\frac{\alpha z_t l_t^{\alpha-1}}{1+r} \frac{1}{\psi_t} - w = - \left( \frac{1}{1 - \lambda^2} \right)^\gamma \gamma \xi_t b_t^\gamma \pi_t^{-\gamma-1} \left( \frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w \right) \quad (\text{S13})$$

We evaluate this first order condition in the steady state:

$$\frac{\alpha z l^{\alpha-1}}{1+r} \frac{1}{\psi} - w = - \left( \frac{1}{1-\lambda^2} \right)^\gamma \gamma \xi b^\gamma \pi^{-\gamma-1} \left( \frac{\alpha z l^{\alpha-1}}{1+r} - w \right) \quad (\text{S14})$$

In the benchmark case we have shown that  $\frac{\alpha z l^{\alpha-1}}{1+r} \frac{1}{\psi} - w = 0$ . Since  $\psi > 1$ , it follows that the last term on the right hand side of (S14),  $\left( \frac{\alpha z l^{\alpha-1}}{1+r} - w \right)$ , is positive. This term represents the increase in profits obtained by increasing labour input in a financially constrained equilibrium. Hence the right hand side of equation (S14) has negative value, meaning that the earning based borrowing reduces the effective cost of labour relative to the benchmark case, increasing labour demand. Intuitively, labour has an additional value in increasing profits, and the firm chooses optimally a level above the level chosen in the benchmark model, but still below the level in the absence of financial frictions.

We derive the first order condition for debt to obtain:

$$b_t = \frac{1}{1+r} \left( \frac{y_t}{1+r} - w l_t \right)^{\frac{\gamma}{\gamma-1}} (1 - \lambda_t^2)^{\frac{\gamma}{\gamma-1}} \left( \frac{1}{\gamma \xi_t} \right)^{\frac{1}{\gamma-1}} \left( 1 - \frac{1}{\psi_t} \right)^{\frac{1}{\gamma-1}}.$$

And we substitute it in (S13) and rearrange obtaining:

$$\frac{\alpha z_t l_t^{\alpha-1}}{1+r} \frac{1}{\psi_t} = w - \Psi_t$$

where

$$\Psi_t \equiv (\gamma \xi_t)^{-\frac{1}{\gamma-1}} \left( \frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w \right) \left( \frac{1}{1+r} \right)^{\frac{\gamma}{\gamma-1}} \left( \frac{y_t}{1+r} - w l_t \right)^{\frac{1}{\gamma-1}} (1 - \lambda_t^2)^{\frac{\gamma}{\gamma-1}} \left( 1 - \frac{1}{\psi_t} \right)^{\frac{\gamma}{\gamma-1}} \quad (\text{S15})$$

We then log linearise the labour first order condition around the steady state:

$$\log z_t - (1 - \alpha) l_t - \log \psi_t = - \frac{1}{w - \Psi} \left( \epsilon_l^\Psi \log l_t + \epsilon_\xi^\Psi \log \xi_t + \epsilon_\psi^\Psi \log \psi_t \right) \quad (\text{S16})$$

where  $\frac{1}{w - \Psi}$  is positive, being equal to the steady state value of the marginal productivity of labour. The definition of  $\Psi_t$  in (S15) implies that, around the steady state, the elasticities of  $\Psi_t$  to  $\xi_t$  and  $\psi_t$ ,  $\epsilon_\xi^\Psi$  and  $\epsilon_\psi^\Psi$ , respectively, are positive, while the elasticity of  $\Psi_t$  to  $l_t$ ,

$\epsilon_l^\Psi$ , can be positive or negative. On the one hand the term  $(\frac{y_t}{1+r} - wl_t)^{\frac{1}{\gamma-1}}$  increases in  $l_t$ , because, as mentioned before, profits  $\frac{y_t}{1+r} - wl_t$  increase in labour around the steady state.

However, then term  $\frac{\alpha z_t l_t^{\alpha-1}}{1+r} - w$  decreases in labour, because of decreasing marginal returns. Intuitively the positive term dominates when the firm is very constrained and labour input is substantially below its unconstrained optimal level. However, as  $l_t$  increases towards such level, eventually  $\Psi_t$  goes to zero, and hence the elasticity  $\epsilon_l^\Psi$  becomes negative.

Rearranging we get:

$$\log l_t = \frac{1}{(1-\alpha) - \frac{1}{w-\Psi}\epsilon_l^\Psi} \log z_t + \frac{\frac{1}{w-\Psi}\epsilon_\xi^\Psi}{(1-\alpha) - \frac{1}{w-\Psi}\epsilon_l^\Psi} \log \xi_t - \frac{1 - \frac{1}{w-\Psi}\epsilon_\xi^\Psi}{(1-\alpha) - \frac{1}{w-\Psi}\epsilon_l^\Psi} \log \psi_t \quad (\text{S17})$$

Consider the elasticity of labour to the productivity shock,  $\frac{1}{(1-\alpha) - \frac{1}{w-\Psi}\epsilon_l^\Psi}$ . As in the benchmark model, it is larger the larger is  $\alpha$ , which reduces the degree to which labour productivity falls when labour demand increases. A positive value of  $\epsilon_l^\Psi$  means more labour input reduces its effective cost by relaxing the borrowing constraint. However, this positive effect vanishes, and eventually  $\epsilon_l^\Psi$  becomes negative, as labour increases, as explained above, so that the elasticity of labour to the productivity shock is always positive and finite. Furthermore, consider the elasticity of labour to  $\psi_t$ . In the term at the numerator,  $1 - \frac{1}{w-\Psi}\epsilon_\xi^\Psi$ , “1” is the direct negative effect of tighter financial frictions on labour input. “ $-\frac{1}{w-\Psi}\epsilon_\xi^\Psi$ ” is the dampening effect representing the fact that, around the steady state, more labour input relaxes the borrowing constraint. As explained above, this dampening factor vanishes as labour approaches the unconstrained level, so that it is never larger than the direct effect. Otherwise, more financial frictions would not affect labour negatively, and hence the firm would not be financially constrained in equilibrium.

Finally, we log-linearise the first order condition for debt, we substitute labour using (S17), and we rearrange obtaining:

$$\log b_t = \left( \frac{\Xi}{1 - \frac{1}{w-\Psi}\epsilon_\xi^\Psi} + \frac{\gamma}{\gamma-1} \frac{\frac{z l^\alpha}{1+r}}{\frac{z l^\alpha}{1+r} - w l} \right) \log z_t - \left( \frac{1}{\gamma-1} - \Xi \right) \log \xi_t + \left( \frac{1}{\gamma-1} \frac{1}{\psi-1} - \Xi \right) \log \psi_t \quad (\text{S18})$$

$$\Xi \equiv \eta \frac{\gamma}{\gamma - 1} \frac{1 - \frac{1}{w-\Psi} \epsilon_{\xi}^{\Psi}}{(1 - \alpha) - \frac{1}{w-\Psi} \epsilon_l^{\Psi}}; \eta \equiv \frac{\alpha \frac{z l^{\alpha-1}}{1+r} - w}{\frac{z l^{\alpha-1}}{1+r} - w} \quad (\text{S19})$$

The term  $\eta$  measures the elasticity of profits to labour which, as argued above, is positive in the steady state. Therefore, following the results shown above, the term  $\Xi$  is also positive, and it represents the effect labour input has, because it relaxes the borrowing constraint, on borrowing. It is easy to see that it increases the elasticity of  $b_t$  to the productivity shock, while it dampens the elasticities to both  $\xi_t$  and  $\psi_t$ . As explained before, for financial frictions to exist in the linearised equilibrium, this dampening effect cannot be larger than the direct effects. To see this, notice that the term  $\eta$  is only positive when the firm is financially constrained, and goes to zero as labour input approaches the unconstrained level. So the dampening effect  $\Xi$  cannot become sufficiently large to completely offset the direct effect and neutralize financial frictions.

It follows that we can represent the linear system (10) as:

$$\begin{aligned} \log b_t &= \epsilon_z^b \log z_t - \epsilon_{\xi}^b \log \xi_t + \epsilon_{\psi}^b \log \psi_t \\ \log l_t &= \epsilon_z^l \log z_t + \epsilon_{\xi}^l \log \xi_t - \epsilon_{\psi}^l \log \psi_t \\ \log y_t &= \log z_t + \alpha \log l_t \end{aligned} \quad (\text{S20})$$

Where the coefficients  $\epsilon_z^b, \epsilon_{\xi}^b, \epsilon_{\psi}^b, \epsilon_z^l, \epsilon_{\xi}^l$  and  $\epsilon_{\psi}^l$  are all positive. Substituting  $\log \psi_t$  using (11) we can derive the matrix  $B$  for the case of earning based borrowing:

$$B = \begin{bmatrix} -(\epsilon_{\xi}^b - \epsilon_{\psi}^b \pi_2) & \epsilon_{\psi}^b \pi_3 & (\epsilon_{\psi}^b \pi_4 + \epsilon_z^b) \\ -(\epsilon_{\psi}^l \pi_2 - \epsilon_{\xi}^l) & -\epsilon_{\psi}^l \pi_3 & (\epsilon_z^l - \epsilon_{\psi}^l \pi_4) \\ -\alpha(\epsilon_{\psi}^l \pi_2 - \epsilon_{\xi}^l) & -\alpha \epsilon_{\psi}^l \pi_3 & (1 + \alpha(\epsilon_z^l - \epsilon_{\psi}^l \pi_4)) \end{bmatrix} \quad (\text{S21})$$

There are three terms,  $\epsilon_{\xi}^b - \epsilon_{\psi}^b \pi_2$ ,  $\epsilon_{\psi}^l \pi_2 - \epsilon_{\xi}^l$  and  $\epsilon_z^l - \epsilon_{\psi}^l \pi_4$  in which a direct effect (the first term), is dampened by an indirect effect (the second term). It is easy to see that this dampening effect is smaller than the direct one, for financial frictions to exist around the steady state. For the term  $\epsilon_{\xi}^b - \epsilon_{\psi}^b \pi_2$ , the reasoning is analogous to the one illustrated in Section S1. For the other two terms, we start by claiming that both  $\pi_2$  and  $\pi_3$  are positive.

For the term  $\epsilon_{\psi}^l \pi_2 - \epsilon_{\xi}^l$ , the first component  $\epsilon_{\psi}^l \pi_2$  implies that a financial friction shock

makes borrowing more costly, and this reduces the demand for labour, as in the benchmark model. However, the same shock also increases the relative desirability of labour input to relax the constraint, which is the second term  $\epsilon_\xi^l$ . This second term is dampening in nature, meaning that it cannot be larger than the direct effect. To see this, notice that if that was the case, then financial frictions would not reduce labour, which would imply  $\epsilon_\xi^\Psi = 0$ , and therefore also  $\epsilon_\xi^l = 0$ . Hence it must be that  $-(\epsilon_\psi^l \pi_2 - \epsilon_\xi^l) < 0$ . A similar reasoning can be used to argue that the term  $\epsilon_z^l - \epsilon_\psi^l \pi_4$  is positive, where  $\epsilon_z^l$  is the direct positive effect of productivity on labour input, while  $-\epsilon_\psi^l \pi_4$  is the dampening effect caused by higher financial frictions  $\psi_t$ . Once again this indirect effect cannot dominate, because it would drive the value of  $\epsilon_\psi^l$  to zero. Given this, to prove Proposition 1 it is sufficient to prove that the coefficients  $\pi_2$ ,  $\pi_3$  are positive, and that  $\pi_4$  is not smaller than  $-\frac{\epsilon_z^b}{\epsilon_\psi^b}$ .

The budget constraint follows equation (S1), where  $\Theta(b_t, \xi_t)$  includes the earning based constraint as defined in (24). We linearise  $\Theta(b_t, \xi_t)$  obtaining:

$$\log \Theta_t = \pi_b \log b_t - \pi_\xi \log \xi_t + \pi_z \log z_t + \pi_l \log l_t$$

Where  $\pi_b$  and  $\pi_\xi$  are positive, as in the benchmark model, while the previous discussion implies that around the steady state more productivity  $z_t$  and more labour input allow more borrowing, and hence also  $\pi_z$  and  $\pi_l$  are positive. Therefore linearising the budget constraint yields:

$$\pi_b \log b_t - \pi_\xi \log \xi_t + \pi_z \log z_t = -\left(\frac{s}{\Theta} - \overline{div} \pi_{div}^s\right) \log s_t + \frac{F}{\Theta} \log \theta_t - \frac{\overline{div} \pi_{div}}{\Theta} \log \psi_t + \left(\frac{wl}{\Theta} - \pi_l\right) \log l_t \quad (\text{S22})$$

Where  $\frac{wl}{\Theta} - \pi_l$  is overall positive, otherwise labour input would not be subject to financial frictions.<sup>2</sup> We then proceed as before, we substitute the labour and debt policy functions in (S22), we solve for  $\psi_t$ , and we compare it to (11) to obtain a solution for the undetermined coefficients  $\pi_2$ ,  $\pi_3$  and  $\pi_4$ :

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<sup>2</sup>Because the increase in wage bill would be lower than the increase in borrowing generated by relaxing the borrowing constraint.

$$\pi_2 = \frac{\pi_b \epsilon_\xi^b + \pi_\xi + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\xi^l}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div\pi_{div}}}{\Theta}}; \quad \pi_3 = \frac{\frac{F}{\Theta}}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div\pi_{div}}}{\Theta}};$$

$$\pi_4 = \frac{\left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_z^l - (\pi_b \epsilon_z^b + \pi_z)}{\pi_b \epsilon_\psi^b + \left(\frac{wl}{\Theta} - \pi_l\right) \epsilon_\psi^l + \frac{\overline{div\pi_{div}}}{\Theta}}$$

It is immediate to see that  $\pi_2 > 0$  and  $\pi_3 > 0$ . Regarding  $\pi_4$ , while its denominator is positive, its numerator depends on a positive term,  $\left(\frac{wl}{\Theta} - \pi_l\right)$ , which reflects the fact that more productivity implies the need for funds to purchase more inputs, and a negative term  $-(\pi_b \epsilon_z^b + \pi_z)$ , which represents the additional borrowing capacity generated by higher profits (for given inputs) when  $z_t$  increases. Intuitively, the negative term is more likely to dominate the smaller is the elasticity  $\alpha$ , which reduces the response of inputs demand to  $z_t$ . To see this, consider that  $\pi_4$  is positive at the steady state if and only if a productivity shock increases pledgeable profits more than the wage bill, or  $\frac{\partial wl}{\partial z} > (1 - \lambda^2) \frac{\partial \pi}{\partial z}$ . We assume for simplicity that the firm is not very financially constrained in the steady state, so that  $\Psi$  is small and we can approximate to zero  $\frac{\partial \Psi}{\partial z}$ . Then it is possible to show that  $\frac{\partial wl}{\partial z} > \frac{\partial \pi}{\partial z}$ , which is a necessary and sufficient condition for  $\pi_4 > 0$ , requires:

$$\frac{w}{w - \Psi} \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}} > \frac{1 - \lambda^2}{2 - \lambda^2} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \quad (\text{S23})$$

In the extreme case that  $\Psi$  is close to zero and profits are fully collateralisable ( $1 - \lambda^2 = 1$ ),  $\pi_4$  is positive as long as  $\alpha$  is larger than 0.5. However, our identifying restrictions are satisfied by much lower values than that. First, because not all profits might be pledged as collateral. For example Drechsel and Kim (2022), in order to match the debt to output ratio observed in the US, using a model with inelastic labour, consider a value of  $1 - \lambda^2$  equal to 0.53, which would lower the minimum value of  $\alpha$  compatible with  $\pi_4 > 0$  to 0.35. Second, condition (S23) is satisfied for a lower value of  $\alpha$  the tighter financial frictions are, in the form of higher values of  $\Psi$  and/or  $\psi$ . Third and more importantly, our identifying conditions are satisfied also when  $\pi_4$  is negative, as long as  $\pi_4 > -\frac{\epsilon_z^b}{\epsilon_\psi^b}$ , where, from (S18), it is easy to see that  $-\frac{\epsilon_z^b}{\epsilon_\psi^b} < -1$ . In other words,  $\alpha$  would have to be so low that a 1%

increase in productivity generates so much additional borrowing that it reduces the intensity of financial frictions more than 1% . Summing up, the term  $B_{13}$  in (S21) would be negative, thus violating proposition 1, only if the elasticity of variable inputs to output ( $\alpha$  in our model) were extremely low, and hence most of the production was done with fixed inputs, which is unrealistic in virtually all industries. Therefore in practice our Assumption 1 is sufficient to satisfy our identification restrictions for realistically chosen values of  $\underline{\alpha}$  and  $\bar{\alpha}$ .

## S2.4 Intra-period borrowing constraint

**Borrowing limited by current assets, as in Buera and Shin (2013).**

Assume that at the beginning of period  $t$  the firm has savings  $s_t$ . Then the three shocks to  $z_t$ ,  $\lambda_t$  (the multiplier factor that determines the borrowing limit) and  $\theta_t$  are realized. The firm needs to borrow if the investment needs for labour  $l_t$  and the fixed cost  $\theta_t F$  are larger than available wealth  $s_t$ . Borrowing is limited by current assets, so that expenditures are constrained to be less than a multiple of  $s_t$ :

$$l_t + \theta_t F \leq \lambda_t s_t \quad (\text{S24})$$

with  $\lambda_t > 1$  and hence the maximum borrowing is  $(\lambda_t - 1)s_t$ . For a micro-foundation of this constraint, see Buera and Shin (2013). After producing, the firm decides savings  $s_{t+1}$  and dividends  $\text{div}_t$  to maximize its intertemporal value:

$$V_t(\mathcal{S}_t) = \max_{l_t, s_{t+1}} \text{div}_t + \eta_t(\lambda_t s_t - l_t - \theta_t F) + \frac{1}{1+r} \frac{1}{\nu} \mathbb{E}_t[V_{t+1}(\mathcal{S}_{t+1})] \quad (\text{S25})$$

subject to the budget constraint:

$$\text{div}_t + \frac{s_{t+1}}{1+r} = s_t + y_t - w l_t \quad (\text{S26})$$

As in the benchmark model, the term  $\nu > 0$  increases the desire to distribute dividends and ensures that financial frictions exist in equilibrium.  $\eta_t$  is the lagrange multiplier associated to constraint S24. The first order condition for savings  $s_{t+1}$  is given by

$$\frac{1}{1+r} + \frac{1}{1+r} \frac{1}{\nu} \left[ \mathbb{E}_t \left( \frac{\partial V_{t+1}}{\partial s_{t+1}} \right) \frac{\partial s_{t+1}}{\partial b_t} \right] = 0 \quad (\text{S27})$$

where  $\frac{\partial s_{t+1}}{\partial b_t} = -1$  and, for the envelope theorem,  $\mathbb{E}_t(\frac{\partial V_{t+1}}{\partial s_{t+1}}) = \mathbb{E}_t[\eta_{t+1}\lambda_{t+1}]$ . Substituting yields:

$$\nu = \mathbb{E}_t(\eta_{t+1}\lambda_{t+1}) \quad (\text{S28})$$

Intuitively, the above condition implies that the gain from distributing dividend early, represented by  $\nu > 0$ , is compensated by the cost of reducing savings and facing future expected financial frictions on the right hand side. If  $\nu > \mathbb{E}_t(\eta_{t+1}\lambda_{t+1})$ , the firm reduces savings  $s_{t+1}$  to increase dividends, but in doing so it increases future expected financial frictions, and the value of  $\mathbb{E}_t(\eta_{t+1}\lambda_{t+1})$  increases until the condition holds with equality. The first order condition for labour yields:

$$\alpha z_t l_t^{\alpha-1} = \eta_t + w \quad (\text{S29})$$

When the constraint is binding and  $\eta_t > 0$ , then labour demand is determined by  $l_t = \lambda a_t$  and the equation above determines the intensity of the constraint,  $\eta_t = \alpha z_t l_t^{\alpha-1} - w > 0$ .

Equation S28 and the assumption of  $\nu > 0$  imply that the constraint is binding in every period with some probability. Equation S24 implies, conditional on savings  $s_t$  and on the productivity shock  $z_t$ , that this probability is weakly monotonously decreasing in  $\lambda_t$ . In other words, a sufficiently strong negative financial frictions shock (a reduction in  $\lambda_t$ ) will cause constraint S24 to bind, reducing debt  $b_t$ , the variable input  $l_t$  and output  $y_t$ . Also, since marginal productivity of labour increases, the reduction in  $y_t$  will be lower than the one in  $l_t$ . Therefore, a shock to  $\lambda_t$ , conditional to being large enough to cause constraint S24 to bind, generates the same sign and inequality restrictions as in the benchmark model.

Regarding the restrictions to identify the shocks to  $\theta_t$  and  $z_t$ , if they are large enough to make the constraint S24 binding, they also satisfy all the restrictions. A negative liquidity shock will increase debt but reduce labour (because of the binding constraint) and output. A positive productivity shock will increase debt, labour and output, and the inequality restrictions will be satisfied following the same reasoning applied in the benchmark model. The reasoning above clarifies that in this model the identification restrictions are valid when the constraint S24 becomes binding as a consequence of a negative shock. A similar reasoning can be used to show that they are also valid when S24 ceases to bind following a positive shock.

Importantly, if the constraint S24 is already binding in the absence of shocks, the iden-

tification restrictions for the financial frictions shock continue to be valid. A reduction in  $\lambda_t$  still reduces, debt, output and labour and increases the output to labour ratio. However, the identification restrictions for the shocks to  $z_t$  and  $\theta_t$  are no longer satisfied. For example, when the constraint binds, a positive productivity shock  $z_t$  is unable to affect borrowing because of the extreme assumptions that first, no uncollateralised borrowing is possible, no matter at what rate, and second, the borrowing limit is unaffected by how productive the firm is. If any of these two extreme assumptions is relaxed, then the identification restrictions for  $z_t$  are satisfied, and it is easy to see that relaxing the first assumption, and allowing some uncollateralised borrowing at higher rates at the margin, would also imply the identification restrictions for the shocks to  $\theta_t$  are satisfied.

### **Intra-period constraint as in Jermann and Quadrini (2012a).**

We provide here the intuition of why the same conclusions can be obtained with the type of intra-period constraint assumed by Jermann and Quadrini (2012a). Adapting to their notation, the firm also uses capital for production as in our extended model with capital,  $y_t = z_t l_t^\alpha k_t^\beta$ . The budget constraint then is:

$$\text{div}_t = s_t + y_t - \theta_t F - w l_t + \frac{b_{t+1}}{1+r} - k_{t+1} + (1-\delta)k_t \quad (\text{S30})$$

Jermann and Quadrini (2012a) assume that all expenses within the period, including fixed capital investment, the reduction in debt (or increase in savings) and the dividend payments, have to be financed with an intraperiod loan  $b_t^{IP}$ , which they show to be equal by construction to revenues,  $b_t^{IP} = y_t$ . Furthermore, both inter- and intra-period borrowing can be secured with capital, but enforcement is imperfect, giving rise to the constraint  $(1 - \xi_t) \left( k_{t+1} - \frac{b_{t+1}}{1+r} \right) \geq b_t^{IP}$ , where  $\xi_t$  is the financial frictions latent variable (see Jermann and Quadrini 2012a for details).<sup>3</sup> Combining this constraint with the budget constraint yields:

$$\frac{1 - \xi_t}{\xi_t} (s_t - \theta_t F - w l_t + (1 - \delta)k_t) \geq y_t \quad (\text{S31})$$

Intuitively, a negative financial frictions shock (an increase in  $\xi_t$ ) reduces borrowing, which is proportional to the left hand side of (S31) and if the shock is large enough to make

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<sup>3</sup>In Jermann and Quadrini (2012a) an increase in  $\xi_t$  denotes a relaxation of financial frictions. To avoid confusion here we maintain the notation adopted earlier in the paper, in which the opposite is true.

the constraint (S31) binding, it implies a reduction in output  $y_t$  and in the variable input  $l_t$ , since the other variables on the left hand side ( $s_t$  and  $k_t$ ) are predetermined. Following the same reasoning adopted above for the case of borrowing limited by the value of assets, one can show that the identification restrictions for the  $\theta_t$  and  $z_t$  shocks are satisfied if these shock cause the constraint to become binding, but not if the constraint was already binding in the absence of shocks. Also in this case the key factor is that this model does not allow the firm to access any amount of more costly uncollateralised borrowing.

## S2.5 Other model extensions

In the model we restrict dividends to be non-negative, and hence we do not allow for equity financing. On the one hand, this assumption is not particularly restrictive especially for small privately owned firms, for which debt is the predominant source of financing. On the other hand, it would be relatively simple to relax it. Since empirically equity issuances are infrequent, one could extend the model to allow for equity financing subject to fixed costs. Then our current identification strategy would likely hold conditional on no equity issuance.

Another characteristic of the model is that the excess cost of credit is the combination of an endogenous variable (debt, or leverage in the model with capital) and a latent variable  $\xi$  which mediates the relation between leverage and excess cost of credit. We assume changes in  $\xi$  to be exogenous, but in reality they might be related to firm fundamentals. For example, a firm that has a sequence of good results for a number of years might be perceived as high quality and the lender will require a lower spread for given leverage. One could address this issue in the model by modifying equation 4 so that  $c_t$  is a more complicated function of lagged endogenous variables.

Finally, our test is based on variable inputs, but in practice a firm might have different degrees of financial frictions across variable and fixed inputs. As demonstrated in section 2.3.1, our financial frictions indicator remains valid in the presence of fixed capital subject to adjustment costs. However, it might be that fixed capital differentially financially constrained than variable capital, and the researcher is specifically interested in measuring the constraints of the former. In this case, one could in principle expand the System 10 to include also the first order condition for fixed capital and use additional restrictions (for example related to the capital/labour ratio) to identify separate financial frictions for different types of inputs.

## S3 Some econometric details

### Limiting distribution panel GMM estimates

In this section we discuss the limiting distribution of the reduced form estimates  $\hat{\mu}$ . We first introduce some additional notation. The commutation matrix  $\mathcal{K}_{m,n}$  is defined such that, for any  $(m \times n)$  matrix  $G$ ;  $\mathcal{K}_{m,n} \text{vec}(G) = \text{vec}(G')$ ; and the  $m^2 \times (m(m+1))/2$  duplication matrix  $\mathcal{D}_m$  is defined such that  $\mathcal{D}_m \text{vech}(F) = \text{vec}(F)$  for a symmetric  $(m \times m)$  matrix  $F$ : Furthermore,  $\mathcal{D}_m^+ = (\mathcal{D}_m' \mathcal{D}_m)^{-1} \mathcal{D}_m'$  and  $\mathcal{L}_m$  is the  $(m(m+1))/2 \times m^2$  elimination matrix defined such that, for any  $(m \times m)$  matrix  $F$ ;  $\text{vech}(F) = \mathcal{L}_m \text{vec}(F)$

To derive the joint limiting distribution of  $\hat{\phi}$  and  $\text{vech}(\hat{\Sigma})$  we impose the following assumptions on the reduced form errors  $u_{i,t} = B\varepsilon_{i,t}$  and the initial conditions.

**Assumption S1.** *We assume that  $u_{i,t}$  is independently and identically distributed across  $i$  and  $t$  and there exists some  $\delta > 0$  such that*

1.  $\mathbb{E}_F(u_{i,t} | W_{i,t}, \dots, W_{i,1}, Y_{i,t-1}, \dots, Y_{i,t-p}) = 0$
2.  $\mathbb{E}_F(u_{i,t} u'_{j,s} | W_{i,t}, \dots, W_{i,1}, Y_{i,t-1}, \dots, Y_{i,t-p}) = \begin{cases} \Sigma & \text{if } i = j, s = t \\ 0 & \text{else} \end{cases}$
3.  $\mathbb{E}_F \|u_{i,t}\|^{4+2\delta} < \infty$
4.  $\max_i \mathbb{E}_F \|X_{i,0}\|^{4+2\delta} < \infty$  and  $\max_i \mathbb{E}_F \|c_i\|^{4+2\delta} < \infty$
5.  $\Sigma_{ZX} = \text{plim}_{N \rightarrow \infty} S_{ZX}$  and  $\Sigma_{ZZ} = \text{plim}_{N \rightarrow \infty} S_{ZZ}$  have full column rank
6.  $\mathbb{E}_F(u_{i,t} u'_{i,t} \otimes u_{i,t} | W_{i,t}, \dots, W_{i,1}, Y_{i,t-1}, \dots, Y_{i,t-p}) = 0$

where  $X_{i,0} = (W'_{i,-p+1}, \dots, W'_{i,0}, Y'_{i,-p}, \dots, Y'_{i,-1})'$ .

To facilitate the characterization of the limiting distribution let

$$Q = \Sigma'_{ZX} \Sigma^{-1}_{ZZ} \Sigma_{ZX} \quad \text{and} \quad S = -\text{plim}_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (u_{i,t} - u_{i,\cdot})(X_{i,t} - X_{i,\cdot})'.$$

The asymptotic distribution of the reduced form estimates  $\hat{\mu} = (\hat{\phi}', \text{vech}(\hat{\Sigma}))'$  is summarized in the following Theorem.

**Theorem S1.** Assume that the data are generated by model (29) and that Assumption S1 holds. We have for  $N \rightarrow \infty$  that

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} N(0, \Omega) , \quad \text{with} \quad \Omega = \begin{bmatrix} \Omega_{\phi\phi} & \Omega'_{\phi\sigma} \\ \Omega_{\phi\sigma} & \Omega_{\sigma\sigma} \end{bmatrix} ,$$

where

$$\begin{aligned} \Omega_{\phi\phi} &= \Sigma \otimes Q^{-1} \\ \Omega_{\phi\sigma} &= -\mathcal{D}_K^+(I_K \otimes S)(\Sigma \otimes Q^{-1}) - \mathcal{D}_K^+ \mathcal{K}_{K,K}(I_K \otimes S)(\Sigma \otimes Q^{-1}) \\ \Omega_{\sigma\sigma} &= \mathcal{D}_K^+ \left[ \frac{1}{T+1} \Lambda_{K^2} + \frac{1}{T(T+1)} (\Sigma \otimes \Sigma)(I_{K^2} \otimes \mathcal{K}_{K,K}) \right] (\mathcal{D}_K^+)' \\ &\quad + \mathcal{D}_K^+(\Sigma \otimes S Q^{-1} S')(\mathcal{D}_K^+)' + \mathcal{D}_K^+(S Q^{-1} S' \otimes \Sigma)(\mathcal{D}_K^+)' \\ &\quad + \mathcal{D}_K^+(\Sigma \otimes S Q^{-1} S') \mathcal{K}'_{K,K}(\mathcal{D}_K^+)' + \mathcal{D}_K^+ \mathcal{K}_{K,K}(\Sigma \otimes S Q^{-1} S')(\mathcal{D}_K^+)' \end{aligned}$$

and  $\Lambda_{K^2} = \mathbb{V}_F(\text{vec}(u_{i,t} u'_{i,t}))$ .

*Proof.* See Theorem 1 in Cao and Sun (2011). □

The important difference with the aggregate time series case is that the limiting covariance between the  $\hat{\phi}$  and  $\text{vech}(\hat{\Sigma})$  is not equal to zero. Therefore we need to take this correlation into account when constructing the confidence sets for the structural coefficients.

The asymptotic variance can be estimated by replacing the population coefficients  $\Sigma$ ,  $Q$ ,  $S$  and  $\Lambda_{K^2}$  by their sample counterparts. For  $\Sigma$  the expression is given in equation (34), while for  $Q$ ,  $S$  these are given by

$$\hat{Q} = S'_{ZX} S_{ZZ}^{-1} S_{ZX} \quad \text{and} \quad \hat{B} = -\frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (u_{i,t} - u_{i,\cdot})(X_{i,t} - X_{i,\cdot})' . \quad (\text{S32})$$

For  $\Lambda_{K^2}$ , some more work is required. In general, the variance matrix  $\Lambda_{K^2}$  depends in a complicated way on the fourth-order multivariate cumulants of  $u_{i,t}$ . Therefore in practice Cao and Sun (2011) suggest to estimate this matrix by

$$\hat{\Lambda}_{K^2} = (\hat{\Sigma} \otimes \hat{\Sigma})(I_{K^2} + \mathcal{K}_{K,K}) . \quad (\text{S33})$$

We followed there approach, noting that in our empirical study little differences are obtained when considering a more general estimate for  $\Lambda_{K^2}$ .

## S4 Simulation study

We use the structural model described in Section 2 to simulate an artificial industry, and draw a panel of  $N = 10000$  firms for  $T = 10$  periods. As explained in Fn.10 in the paper, in the model derived in Section 2.1 we abstract for endogenous exit of firms, to simplify the derivations. Instead for these simulations we consider a slightly modified version of the model in which we allow for endogenous exit to happen in equilibrium. More specifically, the value function can be written as:

$$V_t^{\text{cont}}(\mathcal{S}_t) = \max_{l_t, b_t} (1 + \phi_t) \text{div}_t + \frac{1}{1+r} \frac{1}{\nu} \mathbb{E}_t [V_{t+1}(\mathcal{S}_{t+1})] , \quad (\text{S34})$$

where  $V_t^{\text{cont}}$  is the value conditional on continuing to operate in period  $t$ . The beginning of the period value  $V_t(\mathcal{S}_t)$  is instead given by

$$V_t(\mathcal{S}_t) = d\{1(a_t \geq 0) \max [V_t^{\text{cont}}(\mathcal{S}_t), a_t] - 1(a_t < 0)a_t\} + (1-d)a_t , \quad (\text{S35})$$

With probability  $1-d$ , the firm's technology becomes useless, and the firm liquidates its activity.<sup>4</sup> With probability  $d$  the firm is still productive, but it might go bankrupt and be forced to liquidate if it has insufficient funds to cover the fixed overhead costs of production. We define as  $a \equiv s_t - \theta_t F$  the beginning of the period financial wealth after covering these overhead costs. In (S35),  $1(a_t \geq 0)$  is an indicator function that is equal to zero if  $a_t < 0$  and the firm goes bankrupt. Note that in the calibrated model the beginning of the period financial wealth  $s_t = y_{t-1} - b_{t-1}$  is always positive. That is, the firm always repays the debt. Furthermore, it is also the case that  $a_t$  is almost always positive. The firms engage in precautionary saving to avoid inefficient default that would destroy the positive net present value of their business. Therefore inefficient default with  $a_t < 0$  only happens in equilibrium with around 2% probability.

If  $a_t \geq 0$  and  $\max [V_t^{\text{cont}}(\mathcal{S}_t), a_t] = a_t$ , the firm voluntarily exits because its value net of

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<sup>4</sup>Essentially we assume the stochastic process for productivity  $z_t$  is persistent with probability  $d$  and 0 permanently with probability  $1-d$ .

current wealth ( $V_t^{\text{cont}}(\mathcal{S}_t) - a_t$ ), is negative. This can happen for financial reasons, after a sequence of negative profits which implies excessively high levels of debt, or for efficiency reasons, if productivity becomes too low.

In order to simulate a panel data of firm, we assume that there are  $N$  perfectly competitive firms. We assume the stochastic process for productivity  $z$  to be AR(1) while  $\xi$  and  $\theta$  are assumed to be i.i.d. We solve numerically the model to obtain optimal labour choice  $l_t^*(s_t, \theta_t, z_t, \xi_t)$  and optimal debt  $b_t^*(s_t, \theta_t, z_t, \xi_t)$ . At the beginning of period  $t$  there are  $N$  firms which were active in the previous period.  $N^E$  firms exit after observing the shocks. An equal number of firms enter to keep the total number unchanged. They have identical initial endowment  $a_0$  and initial productivity  $z_0$ . They also draw a value of the shock  $\xi_t$ . Naturally, the values  $a_0$  and  $z_0$  are such that new firms want to continue regardless of  $\xi_t$ .

## Calibration

The parameters of the model are calibrated to match key moments related to the dynamics of productivity and financial variables at the firm level. In other words, we make sure that the simulated panel of firms is as much as possible realistic along the main dimensions of interest. We calibrate all the parameters that are important in determining the level and volatility of productivity, liquidity shocks and financial frictions shocks by matching key firm-level empirical moments. This way we ensure that the dispersion in production opportunities, financial wealth and financial frictions in the model are as much as possible realistic (with the obvious caveat that financial frictions are difficult to measure). Among these, the discount term  $\nu$  affects the desire of firms to distribute dividends and ensures they are borrowers in equilibrium. The chosen value of  $\nu = 1.091$  matches a target leverage of around 25%. The implied discount factor of firms is  $\frac{1}{1+r} \frac{1}{\nu} = 0.877$ . This relatively low value is common in models with financially constrained entrepreneurs that are calibrated to match their distribution of wealth. For example the same parameter is equal to 0.865 in Cagetti and De Nardi (2006). The productivity latent variable  $z$  is an AR(1) process with autoregressive coefficient  $\rho^z$ , while  $\xi$  and  $\theta$  are i.i.d. The unconditional means of  $z$  and  $\theta$ , denoted as  $\mu^z$  and  $\mu^\theta$ , respectively, are normalised to one.  $\rho^z$  matches the autoregressive coefficient obtained from an estimated measure of revenue TFP at the firm level. The standard deviation of the productivity shock  $\varepsilon^z$  matches the cross sectional dispersion in the growth rates of labour, and more specifically the 75th percentile. The standard deviation of the liquidity shock  $\varepsilon^\theta$

matches the the 75th percentile of the cross sectional dispersion in the growth rates of profits over sales. These 3 moments are computed on our dataset of Spanish firms from SABI.<sup>5</sup>

The mean of  $\xi$  and the standard deviation of its innovations  $\varepsilon^\xi$  are difficult to measure empirically, which is precisely the motivation for this paper. For those datasets for which interest rate spreads are available, one can use their distribution to proxy for the distribution of the excess cost of external finance. This is of course an imperfect proxy. On the one hand, empirical spreads reflect in large part also debt riskiness, and therefore are larger than the effective expected cost of debt (because with some probability the firm is going to default and not repay it). This implies that they might overestimate the actual excess cost of finance caused by financial frictions. On the other hand, as we explained before, our estimated financial frictions shock captures not only changes in the excess cost, but also changes in the tightness of the borrowing limit. In this respect, the empirical spread is likely to underestimate the cost of financial frictions. Therefore, even though the empirical spreads are poor measures of financial frictions for a given firm in a given point in time, on average these positive and negative biases partly compensate each other, so that the moments in their distribution might be informative of the moments of the distribution of financial frictions. Thus we use these moments to calibrate our parameters related to the level and volatility of the financial frictions latent variable  $\xi$

More specifically, we calibrate the mean of  $\xi$ ,  $\mu^\xi$ , with the average spread observed on Spanish firm from credit registry data, from Gonzalez and Sy (2023). Furthermore, we calibrate the standard deviation of  $\varepsilon^\xi$ ,  $\sigma^\xi$ , with the dispersion in spread observed for US Compustat firms from Arellano et al. (2019). Finally, we calibrate the fixed cost parameter  $F$  by matching the median ratio between profits and sales for Compustat manufacturing firms, where we measure profits with EBIT (compustat code: ebit).

The other parameters are set as follows: the interest rate  $r$  is set to 4.5%. The initial wealth of firms is set to around 80% of median wealth of incumbents. The output elasticity of labour  $\alpha$  is set to 0.61. The curvature of the excess cost of debt  $\gamma$  is set to 2.<sup>6</sup>

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<sup>5</sup>To reduce in the data some types of firm heterogeneity not represented in the model, we subtract the part of growth rates explained by sector and year fixed effect and we reduce size heterogeneity by including firms between 20 and 250 employees.

<sup>6</sup>In practice,  $\mu^\xi$ ,  $\sigma^\xi$  and  $\gamma$  jointly determine the average cost of financial frictions and their dispersion. We chose to set  $\gamma = 2$ , which helps obtaining a much faster convergence in the solution of the model, and as explained below set the other two parameters to match these two moments.

Table S1: CALIBRATION

|                 | Parameter Value | Target                                   | Data               | Model             |
|-----------------|-----------------|------------------------------------------|--------------------|-------------------|
| $\nu$           | 1.091           | Leverage                                 | 0.261 <sup>1</sup> | 0.22 <sup>6</sup> |
| $\rho^z$        | 0.5             | AR(1) coefficient of estimated RTFP      | 0.5 <sup>2</sup>   | 0.5               |
| $\sigma^z$      | 0.0275          | 75% percentile of labour growth          | 7.4% <sup>2</sup>  | 8.1%              |
| $\mu^\xi$       | 0.005           | Average spread on bank loans             | 3.7% <sup>3</sup>  | 3.51%             |
| $\sigma^\xi$    | 0.75            | Cross sectional dispersion in the spread | 1.1% <sup>4</sup>  | 1.16%             |
| $\sigma^\theta$ | 0.03            | 75% percentile of profits growth         | 3.07% <sup>2</sup> | 3.18%             |
| F               | 14.16           | Profits over Sales                       | 6.2% <sup>5</sup>  | 6.5%              |

*Notes:* Untargeted parameters:  $\mu^z$  and  $\mu^\theta$  are normalised to one.  $\alpha = 0.61$ ;  $\gamma = 2$ ;  $r = 4.5\%$ ;  $s_0 = 25$ , corresponding to 80% of the median value for incumbent firms.

1: US firm level data (Source: Giroud and Mueller 2021); 2: our calculations using Spanish firm level data from SABI; 3: Spanish credit registry (Source: Gonzalez and Sy 2023); 4: Compustat (Source: Arellano et al. 2019); 5: our calculation on manufacturing firms from Compustat; 6: Leverage computed as debt over sales.

## Simulation Results

Our first exercise is to regress the estimated structural shocks on the outcome variables, and to compare the coefficients to those obtained with the true structural shocks. In all regressions we also include the log of beginning of the period financial wealth as control variable. The results are shown in Table S2. Column 1 shows reduced form regressions using the true shocks. These are normalized in order to have similar mean and variance of the estimated shocks, so that the size of the coefficients in column 1 are comparable to those in columns 2-4. Column 1 shows that the shocks explain a substantial part of the variation in the dependent variables.<sup>7</sup> Columns 2-4 in each table show the same regressions, this time using the median, lower bound and upper bound values of the estimated shocks. In all three cases, the estimated shocks also explain a large part of the variation of the dependent variables. Furthermore, the estimated coefficients in columns 2-4 are quite close to those in column 1, especially for the financial frictions shock  $\varepsilon_t^\xi$ , which is the main objective of our analysis, and for the productivity shock  $\varepsilon_t^z$ .

Our second exercise is to verify the validity of our financial frictions indicator  $\mathcal{I}_t^\xi$  defined

<sup>7</sup>The unexplained part is due to the fact that we do not include among the regressors the lagged level of the latent productivity process  $z$ , and to a lesser extent to the approximation induced by the linearization.

Table S2: SIMULATION RESULTS

| Output                             | (1)     | (2)     | (3)     | (4)     |
|------------------------------------|---------|---------|---------|---------|
|                                    | True    | Median  | Lower   | Upper   |
| $\hat{\varepsilon}_{i,t}^{\xi}$    | -0.0350 | -0.0321 | -0.0313 | -0.0324 |
| $\hat{\varepsilon}_{i,t}^{\theta}$ | -0.0011 | -0.0155 | -0.0163 | -0.0171 |
| $\hat{\varepsilon}_{i,t}^z$        | 0.0478  | 0.0500  | 0.0533  | 0.0532  |
| Constant                           | 3.3074  | 3.7280  | 2.7086  | 2.7562  |
| Obs.                               | 80,000  | 80,000  | 80,000  | 80,000  |
| $R^2$                              | 0.7329  | 0.6114  | 0.6887  | 0.6951  |
| Labour                             |         |         |         |         |
|                                    | (1)     | (2)     | (3)     | (4)     |
|                                    | True    | Median  | Lower   | Upper   |
| $\hat{\varepsilon}_{i,t}^{\xi}$    | -0.0571 | -0.0528 | -0.0519 | -0.0518 |
| $\hat{\varepsilon}_{i,t}^{\theta}$ | -0.0017 | -0.0242 | -0.0224 | -0.0241 |
| $\hat{\varepsilon}_{i,t}^z$        | 0.0405  | 0.0421  | 0.0465  | 0.0463  |
| Constant                           | 5.6170  | 6.1110  | 5.1886  | 5.1816  |
| Obs.                               | 80,000  | 80,000  | 80,000  | 80,000  |
| $R^2$                              | 0.7875  | 0.7091  | 0.7586  | 0.7546  |
| Debt                               |         |         |         |         |
|                                    | (1)     | (2)     | (3)     | (4)     |
|                                    | True    | Median  | Lower   | Upper   |
| $\hat{\varepsilon}_{i,t}^{\xi}$    | -0.4404 | -0.4177 | -0.4161 | -0.4241 |
| $\hat{\varepsilon}_{i,t}^{\theta}$ | 0.0054  | 0.0887  | 0.1185  | 0.1171  |
| $\hat{\varepsilon}_{i,t}^z$        | 0.1065  | 0.1175  | 0.1244  | 0.1222  |
| Constant                           | 2.6758  | 2.1284  | 2.8343  | 2.1981  |
| Obs.                               | 80,000  | 80,000  | 80,000  | 80,000  |
| $R^2$                              | 0.8832  | 0.8571  | 0.8253  | 0.8115  |

*Notes:* The table shows the results from regressing output  $\log y_t$ , labour  $\log l_t$  and debt  $\log b_t$  on the true shocks (column (1)) and the estimated structural shocks (columns (2)-(4)). The estimates for the shocks correspond to the median (2), lower (3) and upper (4) bound of the identified set. The log of beginning of the period financial wealth is also included in all regressions as a control variable.

in (26). From our model simulations we can obtain the value of  $\psi_t$ , which measures the relative shadow value of finance and therefore the intensity through which financial frictions affect the real decisions of firms. Therefore, we can measure to what extent, in a realistically calibrated industry, fluctuations in  $\psi_t$  are driven by the financial shock  $\varepsilon_t^\xi$ . In Columns 1 and 2 we show, using linear regressions, the share of variation in  $\psi_t$  explained by each shock. Column 1 considers the true shocks, and Column 2 the median estimated ones. They show that the most important one is by far the financial shock, which explains 73% of the  $\psi_t$  variation in Column 1, and 64.5% in Column 2. The productivity shock only explains around 10% of the variation in  $\psi_t$ . Importantly, this is not because the productivity shock is not very volatile. In fact it is the main driver of output, explaining 52% of its variation, while the financial friction shock only explains 19% of its variation.

The third column does a similar exercise, by directly evaluating the financial frictions indicator. We consider indicators based on the 50% highest values of the three shocks  $\varepsilon^\xi$ ,  $\varepsilon^\theta$ ,  $\varepsilon^z$ , and equal to zero otherwise. In the column, we report the percentage of observations that coincide with the 50% highest values of  $\psi$ . A value around 50% would indicate that the high shock observations are unable to identify firms with high  $\psi$ . This is the case of the  $z$  and  $\theta$  shocks. Conversely, we find that high  $\xi$  observations include 85% of the above median observations of  $\psi$ , and 97% of the highest quartile observations of  $\psi$  (column 4). We also find very similar results if we use the alternative financial frictions indicators described in (42) and (43).

Summing up, these simulation results show that, in the context of the class of models considered, our methodology obtains useful information on the unobservable shocks, and that the indicator  $\mathcal{I}_t^\xi$  is a reliable indicator of financial frictions. Similar results can be obtained when data is simulated from the more elaborate models discussed in Section 2.3.<sup>8</sup>

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<sup>8</sup>In the appendix S2.4 we show that, in a model with earning based constraints, productivity shocks are less positively related to the intensity of financial constraints, because an increase in  $z$  increases desired investment, which increases financial frictions as in the benchmark model, but also it increases the collateral value of earnings, and thus partially relaxes financial frictions. Therefore in a model with earning based borrowing, the financial frictions shock  $\varepsilon^\xi$  would be even relatively more important than  $\varepsilon^z$  in driving the fluctuations in  $\psi$ , than in the benchmark model.

| Table S3: VARIATION IN $\psi$ EXPLAINED BY THE SHOCKS |                       |                 |                     |                     |
|-------------------------------------------------------|-----------------------|-----------------|---------------------|---------------------|
|                                                       | % variation explained |                 | Share $\psi > 50\%$ | Share $\psi > 75\%$ |
|                                                       | True shock            | Estimated shock |                     |                     |
| $\varepsilon^\xi$                                     | 73%                   | 65.6%           | 86%                 | 97%                 |
| $\varepsilon^\theta$                                  | 0.1%                  | 10.8%           | 53%                 | 63%                 |
| $\varepsilon^z$                                       | 9.5%                  | 9.3%            | 59%                 | 41%                 |

*Notes:* In Column 1 we display the  $R^2$  obtained from regressing the relative shadow value of finance  $\psi_t$  on each of the three true shocks individually. In column 2 we do the same exercise using instead the median estimated shocks. In Column 3 we report the percentage of observations for which the indicators that select the 50% firms with highest  $\varepsilon^\xi, \varepsilon^\theta$  and  $\varepsilon^z$  (respectively in rows 1, 2 and 3) include the observations with 50% highest value of  $\psi$ . In column 3 we show the same exercise but this time with the 75% highest values of  $\psi$ .

## S5 Further empirical results

### S5.1 Consistency with narrative indicators

In this section, we compare our indicators of financial frictions with those constructed using narrative methods. In particular, we consider the indicator proposed by Hoberg and Maksimovic (2014). These authors perform an automatic text analysis of the “Liquidity and Capital Resources” section of the 10-K reports of public US firms. They look for a set of keywords that are related to “delay”, and that are relatively close in the text to another set of keywords related to investment plans, and derive a quantitative index, called *delaycon*, that measures the likelihood firms signal the intention to delay their investment plans because of liquidity issues. The index measures the severity of this problem in the cardinal sense, meaning that it can be used to rank firms according to the severity of their constraints (both cross sectionally and over time), and it is publicly available.<sup>9</sup>

Further, the authors refine this indicator with additional information on the firms financing plans. The indicator *debt-delaycon* is high if additional keywords indicate the firms will likely address these liquidity problems by issuing debt. Conversely, the indicator *equity-delaycon* signals the firms will likely address them by issuing equity.

<sup>9</sup>See <http://faculty.marshall.usc.edu/Gerard-Hoberg/MaxDataSite/index.html>. The authors initially define a set of firms identified as financially constrained while the other firms are considered not financially constrained. then they construct a continuous version of this indicator using cosine similarity methods.

In terms of the power of the *delaycon* indicator to detect financially constrained firms, the authors argue that because the keywords related to delay and investment are mentioned in the “Capitalization and Liquidity” subsection, it follows by context that these firms are delaying investment due to liquidity problems. However lacking an explicit causal link from liquidity issues to capital investment decisions, the concern remains that the liquidity problems might be related to timing issues rather than financial constraints that significantly raise the cost of external finance.

In this respect, we think the subindex *equitydelaycon* is a more precise indicator, because the well-known pecking order theory implies that issuing equity is on average more costly than issuing debt (e.g. see, among others, Jermann and Quadrini 2012b). Therefore a firm identified as having liquidity problems in financing investment, and planning to issue equity, is likely to be constrained in issuing debt, which should be correlated to a positive financial frictions shock  $\varepsilon_{i,t}^\xi$ .<sup>10</sup> Conversely, the subindex *debt-delaycon* is potentially problematic for our purpose, since a firm that is able to issue debt to address liquidity problems is not likely to be substantially financially constrained. The validity of the subindex *equitydelaycon* is also highlighted by the authors who report that it is more strongly correlated to their narrative evidence on investment delay than *debt-delaycon*. Furthermore, *equitydelaycon* is positively correlated to the Whited and Wu (2006) index of financial frictions, more so than the main *delaycon* index, while *debt-delaycon* is negative correlated (see Table 2 in Hoberg and Maksimovic (2014)).

Another issue with this indicator is that by construction it might mostly capture firms constrained in fixed capital investment, because of the type of keywords used by the authors, which focus on expansion plans and capital investment (see Hoberg and Maksimovic 2014, page 1321). Therefore, firms that do not have expansion plans but are constrained and forced to reduce their variable inputs such as wages and materials, might not be captured by this indicator.

This is not to say that the narrative indicator *equitydelaycon* is flawed, rather that by design it likely focuses on constraints to fixed capital expansion, differently from our measure,

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<sup>10</sup>Note that in our model, financial frictions shocks increase the shadow value  $(1+\phi_t)$  of obtaining additional equity financing. While we assume for simplicity that equity financing is ruled out by the non-negativity constraint on dividends, the model could be extended by allowing costly equity issuances (see the discussion in Appendix S2.5). Such extended model would further confirm the consistency of our financial frictions indicator with the *equitydelaycon* indicator.

which can potentially capture financial frictions affecting both expanding and contracting firms. We control for this potential difference by performing regressions in which we only include firm-year observations in which firms did not reduce their stock of fixed capital.

In Table S4 we show regression results where the dependent variable is *equitydelaycon* and the main explanatory variable is our benchmark financial frictions indicator  $\widehat{\mathcal{I}}_{i,t}^{\xi}$ . Standard errors are clustered at the firm level, and all regressions include Sector-Year fixed effects. In column 1, we consider the full sample, which comprises all manufacturing firms with at least 3 observations in the 1997-2015 period (the years for which the narrative data is available). In column 2 we consider the sub-sample that excludes firm-year observations in which firms reduced their stock of fixed capital. And in column 3 we consider the complementary sub-sample. The coefficient of  $\widehat{\mathcal{I}}_{i,t}^{\xi}$  is positive and significant for the whole sample, and columns 2 and 3 show that the correlation is entirely driven by periods in which the firms did not reduce their stock of fixed assets, which is consistent with the view that *equitydelaycon* is mainly capturing expansion constraints. Columns 3-6 repeat the same regressions in columns 1-3, adding the same control variables added in Table 1 plus a variable that measures firms size (log of fixed assets). The coefficient of  $\widehat{\mathcal{I}}_{i,t}^{\xi}$  becomes smaller in magnitude but it is still statistically significant for the non-contracting firms in column 5.

Table S5 repeats the analysis of the first three columns of Table S4 for different thresholds to compute  $\widehat{\mathcal{I}}_{i,t}^{\xi}$ . Its coefficient more than doubles when moving from the benchmark indicator (middle panel) to the one that identifies financially constrained firms with those with a financial frictions shock above the 90th percentile (lower panel). One plausible explanation is that in normal times few Compustat firms are financially constrained, and the 90th percentile is a more precise threshold to identify them. In terms of magnitude, the interquartile range of the dependent variable *equitydelaycon* is equal to 0.1. Therefore, with respect to this range, belonging to the most constrained firms according to column 2 (lower panel), increases the value of the narrative indicator by 16%.

## S5.2 Construction of the alternative financial frictions indicators indicators

In this section we briefly describe how we compute the three alternative financial constraints indicators used in the paper. For the Whited and Wu (2006) index, we follow the authors

Table S4: CONSISTENCY WITH NARRATIVE INDICATORS OF FINANCIAL FRICTIONS.

|                               | (1)     | (2)      | (3)     | (4)      | (5)      | (6)      |
|-------------------------------|---------|----------|---------|----------|----------|----------|
| $\widehat{\mathcal{I}}_t^\xi$ | 0.003*  | 0.007*** | 0.000   | 0.002    | 0.006**  | -0.001   |
|                               | (1.663) | (2.630)  | (0.077) | (1.023)  | (2.260)  | (-0.473) |
| $small_t$                     |         |          |         | 0.028*** | 0.026*** | 0.031*** |
|                               |         |          |         | (5.202)  | (4.650)  | (4.486)  |
| $highlev_t$                   |         |          |         | 0.000    | -0.000   | 0.006    |
|                               |         |          |         | (0.107)  | (-0.100) | (1.201)  |
| $lowprod_t$                   |         |          |         | 0.018*** | 0.017*** | 0.021*** |
|                               |         |          |         | (4.958)  | (4.308)  | (4.201)  |
| $\log(fixedassets_t)$         |         |          |         | 0.002    | 0.001    | 0.002    |
|                               |         |          |         | (1.224)  | (0.778)  | (1.137)  |
| $lev_t$                       |         |          |         | 0.001    | -0.000   | 0.001    |
|                               |         |          |         | (1.120)  | (-0.265) | (1.278)  |
| $labp_t$                      |         |          |         | -0.000   | -0.000   | -0.000   |
|                               |         |          |         | (-0.616) | (-0.356) | (-0.715) |
| Obs.                          | 13,697  | 8,147    | 5,550   | 13,557   | 8,063    | 5,494    |
| $R^2$                         | 0.072   | 0.097    | 0.078   | 0.096    | 0.117    | 0.112    |

*Notes:* The table shows the results from regressing a narrative indicator on financial frictions on the indicator computed using our procedure. The dependent variable *equitydelaycon* is a quantitative indicator which is higher the more likely the firm is to delay investment decisions because of liquidity problems, which the firm plan to address them by issuing equity (see Hoberg and Maksimovic (2014) for details). Among the regressors,  $\widehat{\mathcal{I}}_t^\xi$  is a dummy variable equal to one for the upper quartile of the financial friction shock  $\widehat{\varepsilon}_t^\xi$  in year  $t$  and zero otherwise.  $lev_t$  is total debt over fixed assets, and  $labp_t$  is labour productivity (measured as real output divided by number of employees). The variables  $highlev_t$  and  $lowprod_t$ , are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year  $t - 1$ , respectively, and equal to zero otherwise.  $Small_t$  is a dummy variable if the firm belongs to the quartile of smallest firms in period  $t$ , and equal to zero otherwise. All specifications include a constant and sector-year fixed effects. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

and we compute:

$$WW_{i,t} = -0.091 CF_{i,t} - 0.062 DIVPOS_{i,t} + 0.021 TLTD_{i,t} - 0.044 LNTA_{i,t} + 0.102 ISG_{i,t} - 0.035 SG_{i,t},$$

Table S5: CONSISTENCY WITH NARRATIVE INDICATORS OF FINANCIAL FRICTIONS, ALTERNATIVE THRESHOLDS.

| $\widehat{\mathcal{I}}_t^\xi = 1 \text{ if } \varepsilon_t^\xi > 50\%$                     |                     |                     |                    |
|--------------------------------------------------------------------------------------------|---------------------|---------------------|--------------------|
|                                                                                            | (1)                 | (2)                 | (3)                |
| $\widehat{\mathcal{I}}_t^\xi$                                                              | 0.001<br>(0.815)    | 0.003<br>(1.501)    | -0.001<br>(-0.424) |
| Obs.                                                                                       | 13,697              | 8,147               | 5,550              |
| $R^2$                                                                                      | 0.072               | 0.097               | 0.078              |
| $\widehat{\mathcal{I}}_t^\xi = 1 \text{ if } \varepsilon_t^\xi > 75\% \text{ (Benchmark)}$ |                     |                     |                    |
|                                                                                            | (1)                 | (2)                 | (3)                |
| $\widehat{\mathcal{I}}_t^\xi$                                                              | 0.003*<br>(1.663)   | 0.007***<br>(2.630) | 0.000<br>(0.077)   |
| Obs.                                                                                       | 13,697              | 8,147               | 5,550              |
| $R^2$                                                                                      | 0.072               | 0.097               | 0.078              |
| $\widehat{\mathcal{I}}_t^\xi = 1 \text{ if } \varepsilon_t^\xi > 90\%$                     |                     |                     |                    |
|                                                                                            | (1)                 | (2)                 | (3)                |
| $\widehat{\mathcal{I}}_t^\xi$                                                              | 0.012***<br>(3.829) | 0.016***<br>(3.506) | 0.009**<br>(2.173) |
| Obs.                                                                                       | 13,697              | 8,147               | 5,550              |
| $R^2$                                                                                      | 0.073               | 0.099               | 0.080              |

*Notes:* The table shows the results from regressing a narrative indicator on financial frictions on the indicator computed using our procedure. The dependent variable *equitydelaycon* is a quantitative indicator which is higher the more likely the firm is to delay investment decisions because of liquidity problems, which the firm plan to address them by issuing equity (see Hoberg and Maksimovic (2014) for details).  $\widehat{\mathcal{I}}_t^\xi$  is a dummy variable equal to one for the upper quartile of the financial friction shock  $\varepsilon_t^\xi$  in year  $t - 1$  and zero otherwise. All specifications include a constant and sector-year fixed effects. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

where  $CF_{i,t} = (ib_{i,t} + dp_{i,t})/at_{i,t}$  is the ratio of cash flow to total assets;  $DIVPOS_{i,t}$  is an indicator equal to one if the firm pays a positive common dividend per share and zero otherwise;  $TLTD_{i,t} = dl_{i,t}/at_{i,t}$  is the ratio of long-term debt to total assets;  $LNTA_{i,t} = \ln(at_{i,t})$  is the natural logarithm of total assets;  $SG_{i,t} = sale_{i,t}/sale_{i,t-1} - 1$  is firm sales growth; and  $ISG_{i,t}$  is industry sales growth, computed as the growth rate of the sum of

revenues across all firms sharing the same 3-digit SIC code in year  $t$ .

For the Kaplan and Zingales (1997) index we use the linear specification by Lamont et al. (2001):

$$KZ_{i,t} = -1.001909 \frac{ib_{i,t} + dp_{i,t}}{ppent_{i,t-1}} + 0.2826389 Q_{i,t} + 3.139193 \frac{dltt_{i,t} + dlc_{i,t}}{dltt_{i,t} + dlc_{i,t} + seq_{i,t}} \\ - 39.3678 \frac{dvc_{i,t} + dvp_{i,t}}{ppent_{i,t-1}} - 1.314759 \frac{che_{i,t}}{ppent_{i,t-1}},$$

where  $Q_{i,t} = (at_{i,t} + prcc\_f_{i,t} \cdot csho_{i,t} - ceq_{i,t} - txdb_{i,t})/at_{i,t}$  is the proxy for Tobin's  $Q$ .  $ib$  is income before extraordinary items;  $dp$  is depreciation and amortisation;  $ppent$  is net property, plant and equipment;  $dltt$  is long-term debt;  $dlc$  is debt in current liabilities;  $seq$  is stockholders' equity;  $dvc$  is dividends on common stock;  $dvp$  is dividends on preferred stock;  $che$  is cash and short-term investments;  $at$  is total assets;  $prcc\_f$  is fiscal-year closing share price;  $csho$  is common shares outstanding;  $ceq$  is common equity;  $txdb$  is deferred taxes.

For the Hadlock and Pierce (2010) indicator, we compute the size-age (SA) index as:

$$HP_{i,t} = -0.737 Size_{i,t} + 0.043 Size_{i,t}^2 - 0.040 Age_{i,t},$$

where  $Size_{i,t}$  is the natural logarithm of firm total assets expressed in 2004 dollars, and  $Age_{i,t}$  is the number of years the firm has been listed in Compustat (measured as the cumulative count of years with a non-missing stock price). To convert nominal assets into 2004 dollars we use the annual average of the BLS Consumer Price Index for All Urban Consumers. As in Hadlock and Pierce (2010),  $Size_{i,t}$  is capped at  $\ln(4,500)$  (with total assets in millions of dollars) and  $Age_{i,t}$  is capped at 37 years, so that the index is not driven by very large or very old firms.

For all three indexes higher values correspond to firms that are more likely to be financially constrained. In some applications we use directly the value of the indexes, namely in the regressions shown in Figure 1 in the paper and in panels B-D in Figure S1 in the appendix, while in all the other applications we use binary indicators  $\mathcal{I}_{i,t}^{WW Index}$ ,  $\mathcal{I}_{i,t}^{KZ Index}$  and  $\mathcal{I}_{i,t}^{HP Index}$ , which are equal to one when the value of the corresponding continuous index lies in the top quartile of the within-year distribution, and to zero otherwise.

Table S6: FINANCIAL FRICTIONS IN 2007 AND EMPLOYMENT CONTRACTION IN 2008.  
ALTERNATIVE THRESHOLDS TO COMPUTE THE FINANCIAL FRICTIONS INDICATOR.

|                              | (1)                   | (2)                   | (3)                   |
|------------------------------|-----------------------|-----------------------|-----------------------|
|                              | > 50%                 | > 75% (Benchmark)     | > 90%                 |
| $\log(l_{t-1})$              | 0.895***<br>(96.684)  | 0.896***<br>(99.962)  | 0.896***<br>(101.229) |
| $\log(l_{t-1}) * Gr$         | -0.003<br>(-0.949)    | -0.006<br>(-1.577)    | -0.006<br>(-1.546)    |
| $\mathcal{I}_{t-1}^\xi$      | -0.002<br>(-0.489)    | 0.003<br>(0.575)      | -0.001<br>(-0.171)    |
| $\mathcal{I}_{t-1}^\xi * Gr$ | -0.041***<br>(-3.116) | -0.067***<br>(-3.967) | -0.088***<br>(-3.441) |
| Constant                     | 0.151***<br>(9.736)   | 0.146***<br>(10.087)  | 0.148***<br>(10.504)  |
| Obs.                         | 6,710                 | 6,710                 | 6,710                 |
| $R^2$                        | 0.858                 | 0.858                 | 0.858                 |
| Number of firm               | 506                   | 506                   | 506                   |

*Notes:* The table studies the differential effects of the great recession on financially constrained firms. In column 1, Firm fixed effects and Sector-Year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year  $t$ ,  $\log(l_t)$ . Among the regressors,  $\widehat{\mathcal{I}}_{t-1}^\xi$  is a dummy variable equal to one for the firms with largest value of the financial friction shock  $\hat{\varepsilon}_{t-1}^\xi$  in year  $t - 1$  and zero otherwise. In columns 1, 2 and 3 the threshold value of  $\hat{\varepsilon}_{t-1}^\xi$  is the 50th, 75th and 90th percentile, respectively.  $Gr$  is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

### S5.3 Robustness of the main results in the paper

In this section we provide some robustness checks of the main results shown in Section 4.2. Panels A-D of Figure S1 replicate the results in panels A-D of Figure 1 but without normalising the indicators to having mean zero and unit standard deviation. Panels E-H repeat the same analysis but using binary indicators.  $\mathcal{I}_{i,t}^{WWIndex}$ ,  $\mathcal{I}_{i,t}^{KZIndex}$  and  $\mathcal{I}_{i,t}^{HPIndex}$  correspond to the Whited and Wu (2006), Kaplan and Zingales (1997) and Hadlock and Pierce (2010) index, respectively. They are equal to one if the firm is among the 25% with the highest value of the index in that year, and equal to zero otherwise.

Table S6 repeats the estimation in column 1 of Table 1, but varying the threshold  $\tau$  in 41. Table S7 repeats the same estimations but selecting different values of the identified set. Results are qualitatively similar regardless on whether we compute the indicator  $\widehat{\mathcal{I}}_{i,t-1}^\xi$  using the upper or lower bound values of the identified set for  $\hat{\varepsilon}_{t-1}^\xi$ .

Table S8 replicates the exercise in Table 1 but adding other financial frictions indica-

Figure S1: Covenant violations — indicators not rescaled and binary indicators

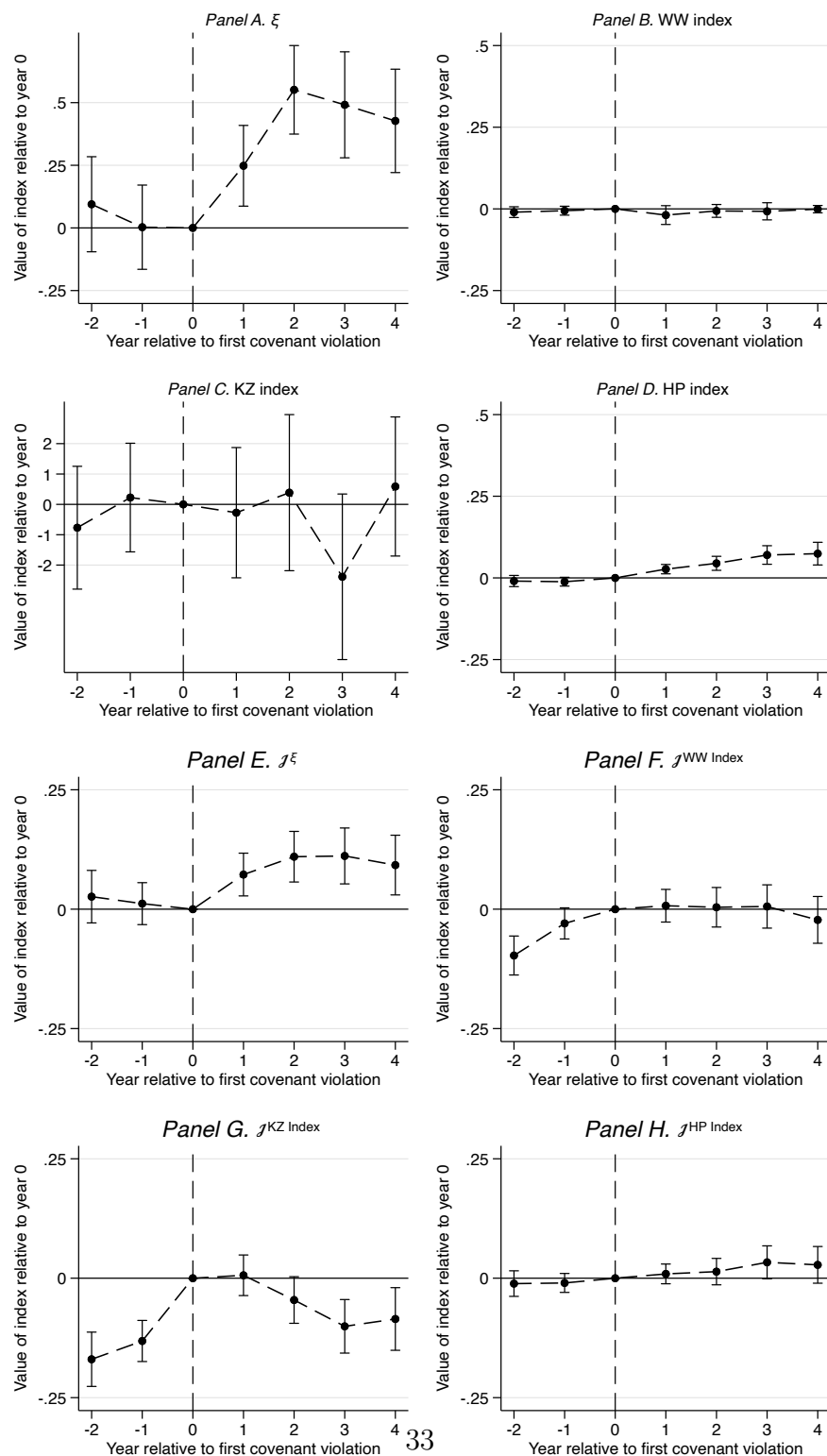


Table S7: FINANCIAL FRICTIONS IN 2007 AND EMPLOYMENT CONTRACTION IN 2008, ALTERNATIVE VALUES OF THE IDENTIFIED SET

|                              | (1)                                                | (2)                                                       | (3)                                                |
|------------------------------|----------------------------------------------------|-----------------------------------------------------------|----------------------------------------------------|
|                              | lower bound value of $\hat{\varepsilon}_{t-1}^\xi$ | median value of $\hat{\varepsilon}_{t-1}^\xi$ (Benchmark) | upper bound value of $\hat{\varepsilon}_{t-1}^\xi$ |
| $\log(l_{t-1})$              | 0.895***<br>(99.422)                               | 0.896***<br>(99.962)                                      | 0.895***<br>(97.772)                               |
| $\log(l_{t-1}) * Gr$         | -0.003<br>(-0.845)                                 | -0.006<br>(-1.577)                                        | -0.005<br>(-1.477)                                 |
| $\mathcal{I}_{t-1}^\xi$      | -0.002<br>(-0.407)                                 | 0.003<br>(0.575)                                          | -0.004<br>(-0.627)                                 |
| $\mathcal{I}_{t-1}^\xi * Gr$ | -0.052***<br>(-3.285)                              | -0.067***<br>(-3.967)                                     | -0.039**<br>(-2.103)                               |
| Obs.                         | 6,735                                              | 6,735                                                     | 6,735                                              |
| $R^2$                        | 0.856                                              | 0.857                                                     | 0.856                                              |
| Number of firm               | 508                                                | 508                                                       | 508                                                |

*Notes:* The table studies the differential effects of the great recession on financially constrained firms. In column 1, Firm fixed effects and Sector-Year fixed effects are included in all specifications. The dependent variable is the logarithm of the number of employees in year  $t$ ,  $\log(l_t)$ . Among the regressors,  $\hat{\mathcal{I}}_{t-1}^\xi$  is a dummy variable equal to one for the firms with largest value of the financial friction shock  $\hat{\varepsilon}_{t-1}^\xi$  in year  $t - 1$  and zero otherwise. In columns 1, 2 and 3 the value of  $\hat{\varepsilon}_{t-1}^\xi$  corresponds to the lower bound, median, and upper bound of the identified set, respectively.  $Gr$  is a dummy variable equal to 1 for year 2008 and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

tors commonly used in the literature.  $\mathcal{I}_{i,t}^{WWIndex}$ ,  $\mathcal{I}_{i,t}^{KZIndex}$  and  $\mathcal{I}_{i,t}^{HPIIndex}$  correspond to the Whited and Wu (2006), Kaplan and Zingales (1997) and Hadlock and Pierce (2010) index, respectively. They are equal to one if the firm is among the 25% with the highest value of the index in that year, and equal to zero otherwise.

Table S9 replicates the exercise in Table 1 using the large sample of Spanish firms from SABI, which is representative of the population of firms above 5 employees. One limitation of this sample is that it is only available from 2000 (some years before 2000 are available, but with very limited coverage). Since we use two lags to compute the shocks, we can only include the years from 2002 to 2007 for the independent variables. We focus on a balanced sample of firms present in the whole period, for a total of 26.868 firms. Table S9 shows that the coefficient of  $\mathcal{I}_{t-1}^\xi$  is negative and significant, indicating that over the sample period, firms that are more financially constrained in year  $t$  reduce their employment in year  $t + 1$  more than the other firms. This result, which is in contrast to what we found for Compustat in Table 1, is likely due to the fact that we are analyzing predominantly small firms for which financial shocks are more persistent than for large ones also in normal times, not only during a financial crisis.

Table S8: FINANCIAL FRICTIONS IN 2007 AND EMPLOYMENT CONTRACTION IN 2008 - OTHER INDICATORS

| VARIABLES                           | (1)                   | (2)                   | (3)                   | (4)                   |
|-------------------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\log(l_{t-1})$                     | 0.891***<br>(102.903) | 0.885***<br>(102.936) | 0.885***<br>(102.547) | 0.884***<br>(99.347)  |
| $\log(l_{t-1}) * Gr$                | 0.005<br>(1.131)      | 0.004<br>(0.738)      | 0.004<br>(0.754)      | 0.005<br>(1.057)      |
| $\mathcal{I}_{t-1}^{\xi}$           | -0.004<br>(-0.680)    | -0.004<br>(-0.657)    | -0.004<br>(-0.683)    | -0.004<br>(-0.674)    |
| $\mathcal{I}_{t-1}^{\xi} * Gr$      | -0.059***<br>(-3.142) | -0.058***<br>(-3.062) | -0.062***<br>(-3.251) | -0.063***<br>(-3.269) |
| $\mathcal{I}_{t-1}^{WWIndex}$       |                       | -0.043***<br>(-3.558) | -0.037***<br>(-3.044) | -0.036***<br>(-3.001) |
| $\mathcal{I}_{t-1}^{WWIndex} * Gr$  |                       | -0.018<br>(-0.679)    | -0.013<br>(-0.476)    | -0.027<br>(-0.944)    |
| $\mathcal{I}_{t-1}^{KZIndex}$       |                       |                       | -0.031***<br>(-4.458) | -0.031***<br>(-4.452) |
| $\mathcal{I}_{t-1}^{KZIndex} * Gr$  |                       |                       | -0.009<br>(-0.577)    | -0.009<br>(-0.606)    |
| $\mathcal{I}_{t-1}^{HPIIndex}$      |                       |                       |                       | -0.007<br>(-0.467)    |
| $\mathcal{I}_{t-1}^{HPIIndex} * Gr$ |                       |                       |                       | 0.032<br>(1.205)      |
| Constant                            | 0.064<br>(0.930)      | 0.119*<br>(1.655)     | 0.139*<br>(1.932)     | 0.144*<br>(1.955)     |
| Observations                        | 6,710                 | 6,710                 | 6,710                 | 6,710                 |
| R-squared                           | 0.860                 | 0.861                 | 0.862                 | 0.862                 |
| Number of firm                      | 506                   | 506                   | 506                   | 506                   |
| Firm FE                             | YES                   | YES                   | YES                   | YES                   |
| Sector*Year FE                      | YES                   | YES                   | YES                   | YES                   |

Robust t-statistics in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

*Notes:* The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects, sector-year fixed effects, and the same control variables included in Column 4 of Table 1 are included in all specifications. The dependent variable is the logarithm of the number of employees in year  $t$ ,  $\log(l_t)$ . Among the regressors,  $\hat{\mathcal{I}}_{t-1}^{\xi}$  is a dummy variable equal to one for the upper quartile of the financial friction shock  $\hat{\varepsilon}_{t-1}^{\xi}$  in year  $t - 1$  and zero otherwise.  $\hat{\mathcal{I}}_{t-1}^{WWIndex}$  is a dummy variable equal to one for the upper quartile of the Whited and Wu (2006) indicator and zero otherwise.  $\hat{\mathcal{I}}_{t-1}^{KZIndex}$  is a dummy variable equal to one for the upper quartile of the Kaplan and Zingales (1997) indicator and zero otherwise.  $\hat{\mathcal{I}}_{t-1}^{HPIIndex}$  is a dummy variable equal to one for the upper quartile of the Hadlock and Pierce (2010) indicator and zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Nonetheless, we also find that the Coefficient of  $\mathcal{I}_{t-1}^{\xi} * Gr$  is also negative and significant in all specifications, indicating that this negative relation was even stronger between year 2007 and 2008, thus confirming the results obtained for the Compustat sample.<sup>11</sup>

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<sup>11</sup>Note that, due to the data limitation explained above, the SABI sample only starts in 2002, and hence it could be that the coefficient of  $\mathcal{I}_{t-1}^{\xi} * Gr$  is in part driven by the fact that we compare 2007 with a limited number of years in which there was fast growth in Spain. Conversely, for the Compustat sample, we avoid this problem by focusing on a much longer time series and by performing several placebo experiments, as explained in Section 4.2.2.

Table S9: FINANCIAL FRICTIONS IN 2007 AND EMPLOYMENT CONTRACTION IN 2008 - SMALL AND MEDIUM SPANISH MANUFACTURING FIRMS.

|                                      | (1)                    | (2)                    | (3)                    | (4)                    | (5)                    |
|--------------------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\log(l_{t-1})$                      | 0.569***<br>(96.175)   | 0.563***<br>(94.756)   | 0.576***<br>(96.053)   | 0.642***<br>(107.877)  | 0.621***<br>(103.088)  |
| $\log(l_{t-1}) * Gr$                 | 0.002<br>(1.220)       | 0.000<br>(0.010)       | -0.002<br>(-1.487)     | -0.006***<br>(-3.923)  | -0.007***<br>(-4.297)  |
| $\mathcal{I}_{t-1}^{\xi}$            | -0.019***<br>(-15.770) | -0.021***<br>(-17.308) | -0.020***<br>(-16.780) | -0.018***<br>(-14.434) |                        |
| $\mathcal{I}_{t-1}^{\xi} * Gr$       | -0.042***<br>(-13.134) | -0.042***<br>(-13.168) | -0.040***<br>(-12.620) | -0.034***<br>(-10.655) |                        |
| $\mathcal{I}_{t-1}^Z$                |                        | -0.021***<br>(-17.944) | -0.016***<br>(-13.493) | 0.005***<br>(3.959)    |                        |
| $\mathcal{I}_{t-1}^Z * Gr$           |                        | -0.037***<br>(-11.467) | -0.029***<br>(-8.779)  | -0.011***<br>(-3.390)  |                        |
| $Small_{t-1}$                        |                        |                        | -0.037<br>(-0.798)     | -0.046<br>(-0.962)     | -0.039<br>(-0.803)     |
| $Small_{t-1} * Gr$                   |                        |                        | 0.004<br>(1.023)       | 0.002<br>(0.567)       | 0.001<br>(0.267)       |
| $Highlev_{t-1}$                      |                        |                        | 0.001<br>(0.756)       | 0.000<br>(0.004)       | -0.001<br>(-0.460)     |
| $Highlev_{t-1} * Gr$                 |                        |                        | -0.008**<br>(-2.457)   | -0.008*<br>(-1.681)    | -0.009**<br>(-2.098)   |
| $Lowprod_{t-1}$                      |                        |                        | -0.046***<br>(-19.216) | -0.015***<br>(-5.396)  | -0.014***<br>(-5.161)  |
| $Lowprod_{t-1} * Gr$                 |                        |                        | -0.038***<br>(-11.761) | -0.014***<br>(-3.429)  | -0.013***<br>(-3.334)  |
| $labp_{t-1}$                         |                        |                        |                        | 0.141***<br>(28.097)   | 0.138***<br>(30.433)   |
| $labp_{t-1} * GR$                    |                        |                        |                        | 0.021***<br>(7.767)    | 0.022***<br>(8.361)    |
| $lev_{t-1}$                          |                        |                        |                        | -0.002<br>(-1.221)     | -0.015***<br>(-8.968)  |
| $lev_{t-1} * GR$                     |                        |                        |                        | -0.003<br>(-1.395)     | -0.004**<br>(-2.100)   |
| $\hat{\varepsilon}_{t-1}^{\xi}$      |                        |                        |                        |                        | -0.018***<br>(-21.214) |
| $\hat{\varepsilon}_{t-1}^{\xi} * Gr$ |                        |                        |                        |                        | -0.021***<br>(-12.517) |
| Obs.                                 | 134,272                | 134,272                | 134,272                | 134,272                | 134,272                |
| $R^2$                                | 0.355                  | 0.360                  | 0.366                  | 0.380                  | 0.385                  |
| Number of firm                       | 24,868                 | 24,868                 | 24,868                 | 24,868                 | 24,868                 |

*Notes:* The table shows the differential effects of the great recession on financially constrained firms. Firm fixed effects and sector-year fixed effects (2-digit NACE sectors) are included in all specifications. The dependent variable is the logarithm of the number of employees in year  $t$ ,  $\log(l_t)$ . Among the regressors,  $\widehat{\mathcal{I}}_{t-1}^{\xi}$  is a dummy variable equal to one for the upper quartile of the financial friction shock  $\hat{\varepsilon}_{t-1}^{\xi}$  in year  $t - 1$  and zero otherwise.  $\widehat{\mathcal{I}}_{t-1}^Z$  is a dummy variable equal to one for the lower quartile of the productivity shock  $\hat{z}_{t-1}$  in year  $t - 1$  and zero otherwise.  $Gr$  is a dummy variable equal to 1 for year 2008 and equal to zero otherwise.  $lev_{t-1}$  is total debt over fixed assets, and  $labp_{t-1}$  is labour productivity (measured as real output divided by number of employees). The variables  $highlev_{t-1}$  and  $lowprod_{t-1}$ , are equal to one for the 25% firm-year observations with highest leverage and lower labour productivity in year  $t - 1$ , respectively, and equal to zero otherwise.  $Small_{t-1}$  is a dummy variable if the firm belongs to the quartile of smallest firms in period  $t - 1$ , and equal to zero otherwise. Standard errors are clustered at the firm level. Robust t-statistics are given in parentheses and \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

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